# 3.9) Derivatives of Inverses

day 39

on Tuesday, we were supposed to see the derivative of the inverse of a function (at a corresponding point) is the Reciprocal of the original derivative.

Now the math....

Oct 24-7:22 AM

day 39

Inverse functions

Key idea: switch x & y.

Importance: Once we sweat & figure out how to get one value from another (in a specific relationship)

THE VERY NEXT QUESTION IS how do I do that backwards?

Oct 24-7:22 AM

day 39

IF

$f(g(x)) = x$ for every x in the domain of $g(x)$, and

$g(f(x)) = x$ for every x in the domain of $f(x)$, then

$g(x)$ and $f(x)$ are INVERSE FUNCTIONS

and we write

$g(x) = f^{-1}(x)$ and $f(x) = g^{-1}(x)$.

Oct 24-7:22 AM

day 39

$$\frac{d}{dx}\left[ f\left(f^{-1}(x)\right) = x \right]$$

$$f'\left(f^{-1}(x)\right) \cdot \frac{d}{dx}\left(f^{-1}(x)\right) = 1$$

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'\left(f^{-1}(x)\right)}$$

Oct 24-7:22 AM

day 39

$e^x$ and $\ln(x)$ are inverses.

$10^x$ and $\log_{10}(x)$ "

$2^x$ and $\log_2(x)$ "

$$e^{\ln x} = x$$

$$\ln(e^x) = x$$

$$e^{\ln x} \cdot \frac{d}{dx}(\ln x) = 1$$

$$\frac{d}{dx}(\ln x) = \frac{1}{e^{\ln x}} = \boxed{\frac{1}{x}}$$

day 39

but wait! there's more!

look at $2^x = (e^{\ln 2})^x = e^{x\ln 2}$

$(a^m)^n = a^{m \cdot n}$

$\frac{d}{dx}(e^{x\ln 2}) = (e^{x\ln 2}) \cdot \frac{d}{dx}(x\ln 2)$

$= e^{x\ln 2} \cdot (\ln 2)$

$\frac{d}{dx}(2^x) = 2^x \cdot (\ln 2)$

Oct 24-7:22 AM

# da Rulz

day 39

p.380

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(e^x) = e^x \Leftrightarrow$ * $\frac{d}{dx}(b^x) = b^x \cdot \ln b$

* $\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

Similarly

* $\frac{d}{dx}(\log_b x) = \frac{1}{\ln b \cdot x}$

Oct 24-7:22 AM

day 39

$$\frac{d}{dx} \log_b x$$

$$\log_b x = y$$

$$\ln\left( b^y = x \right)$$

$$\ln(b^y) = \ln x$$

$$\frac{d}{dx}\left( y \ln b = \ln x \right)$$

$$\ln b \cdot \frac{d}{dx}(y) = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{(\ln b)x}$$

So
$$\frac{d}{dx}\left(\log_b x\right) = \frac{1}{\ln(b)x}$$

day 39

$a^m \cdot a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^{-m} = \frac{1}{a^m}$

$a^0 = 1$

$a^1 = a$

$a^{\frac{1}{m}} = \sqrt[m]{a}$

$(abc)^m = a^m b^m c^m$

$\left[\frac{a^{m_1} b^{m_2} c^{m_3}}{d^{m_4} e^{m_5}}\right]^n = \frac{a^{nm_1} b^{nm_2} c^{nm_3}}{d^{nm_4} e^{nm_5}}$

$\log(ab) = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log a^m = m \log a$

$e^{\ln a} = a \;,\; \ln e^a = a$

$\log_b a = x \Leftrightarrow b^x = a$

Oct 24-7:22 AM